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ABSTRACT

The assertion that productivity is declining in education, economists' work on educational production functions, and definitions of efficiency and productivity are discussed in three reports on the economics of education. The measurement and aggregation problems in the determination of productivity changes in education are explained, and three productivity studies are described. Productivity declines are attributed to the lack of incentive for efficiency improvement in public schools, and the fact that education is intrinsically high in labor intensity but low in capital. A report on the literature on educational production functions describes micro-level functions as the learning of relatively simple items, meso-level functions as simple regressions involving student achievement test scores and background variables, and macro-level functions as simulation models which project the number of enrollees, graduates, and dropouts as a function of budget levels, dropout rates, and repetition rates. Relevant articles listed for each level of production, and research directions based on level boundaries are described. In an evaluation of the efficiency and productivity of educational systems, production possibilities, economic efficiency, technical efficiency, and efficiency in the absence of output prices are defined. Cost-benefit and cost-effectiveness analyses are described as a means for determining how gain and optimal levels of inputs and outputs vary with price. (Author/JAC)

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RESEARCH MEMORANDUM

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NOTES ON TOPICS IN THE ECONOMICS OF EDUCATION

Dean T. Jamison

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Educational Testing Service
Princeton, New Jersey
April 1976

TM006 658

Foreword

This Research Memorandum includes three short pieces that I have prepared on various aspects of the economics of education. The first, "Short Notes on Productivity Change in Education", presents a short assessment of the assertion that productivity is declining in education; I originally prepared it while working as a consultant to the NIE. The second piece, "Notes on Educational Production Functions", provides an overview of a lecture I gave in the Spring of 1975 at the Woodrow Wilson School. In it I attempt to place economists' work on educational production functions into the context of a broader range of work. The third piece is slightly more technical. It provides definitions of the terms 'productivity' and 'efficiency' that accord reasonably well with current economic usage. As this usage is occasionally ambiguous, the definitions presented here may be unsatisfactory to some economists; they do, nonetheless, constitute a consistent set of related definitions, and indicate how those definitions can be applied in education. A shorter, less technical version of this third piece appeared in Anderson, Ball, and Murphy's Encyclopedia of Educational Evaluation.

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I. SHORT NOTES ON PRODUCTIVITY CHANGE IN EDUCATION

1. The term 'productivity' refers to the amount of output per unit of input of some system. Such a concept is relatively straightforward when there are simple, easily measured inputs and outputs; the number of apples picked per laborer per hour would be an example. In general, though, the system under consideration will have many inputs and many outputs and one has the problem of aggregating apples and oranges to get a single output measure that can be divided by a single, aggregated input measure. The most complicated version of this task is to assess aggregate input and output for the economy as a whole. Continued effort on the part of economists has resulted in measures of the year by year aggregate input and output of the private sector of the U.S. and other economies for much of this century. During the period 1919-66 the aggregate output of the private sector of the U.S. economy grew at a rate of about 3.25% per annum; during the latter years of this period the rate was higher; during the early years, lower. During the same period, the growth rate of the aggregate capital and labor input was about 1.15% per year. The higher growth rate of output than of input is generally attributed to a growth in productivity in the economy of 2.1% per year, on average, over the period.[1]

2. Government service sectors, such as education, have experienced lower rates of productivity growth to the extent that this growth has been possible to measure. Measurement and aggregation problems are more difficult in assessing productivity changes in education than they are for the private sector, primarily because there exists no readily applicable price system to weight the various cognitive and social/affective outputs of the educational system into an aggregate output measure. Nonetheless a

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[1] For a survey of research on productivity trends see M. I. Nadiri, "Some approaches to the theory and measurement of total factor productivity: A survey", Journal of Economic Literature, December, 1970, 1137-1177. The numbers used here are drawn from Nadiri's paper. Some economists have argued that inputs, 'properly' measured, have grown more rapidly than 1.15% per year and that, therefore, productivity increases are lower than the reported 2.1%. The current concensus seems to support a figure close to the 2.1% figure used here.

number of studies exist that, while they occasionally need to substitute common sense for precise measurement, do give an idea of productivity changes in education. Three such studies are reviewed below.

3. June O'Neill (now with the staff of the Council of Economic Advisors) prepared a study for the Carnegie Commission on Higher Education that provides evidence concerning productivity change in higher education in the U.S.[2] For her output measure she used credit hours granted by colleges and universities over the period 1930 to 1967. During this period output rose at a rate of 5.0% per year while total inputs rose at a rate of 5.1 percent per year. This results in a productivity decline of .1% per year. O'Neill well recognized that incorporating hitherto unmeasured changed in the 'quality' of a credit hour would provide a different measure of output and, perhaps, would indicate a different degree of productivity change. But using the output measure she had available, productivity has been stagnant in U.S. higher education.

4. No studies have addressed directly the assessment of productivity change in U.S. elementary and secondary education. Nevertheless, in a recent Federal Government study of Productivity and Efficiency in Education,[3] there is information on real input use per unit of output that is very much like a productivity measure. That report claimed that 'real resource use'[4] per pupil in public elementary and secondary schools rose at a rate of 2.25% per year over the period 1949-1969. Assuming that the quality of a year of elementary and secondary education has remained unchanged,[5] this would imply a productivity decline of 2.25% per year.

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[2]See J. O'Neill's Resource Use in Higher Education (Carnegie Commission on Higher Education, 1971) and her "Productivity Trends in Higher Education", in J. Froomkin, D. Jamison, and R. Radner (Eds.), Education as an Industry, New York: National Bureau of Economic Research and Columbia University Press, forthcoming in 1976.

[3]See Productivity and Efficiency in Education, a report by the Education Panel to the Committee on Automation Opportunities in the Service Areas, Federal Council for Science and Technology, December, 1972. (Draft)

[4]Real resource use is not the same as cost principally because of the rise of teacher salaries relative to other wages. The inflation adjusted increase in cost per student per year over this period was 4.5% per year; without adjusting for inflation, the increase in cost was 7.2%.

[5]While an intuitive judgment of this is hard to make in the absence of longitudinal data from something like the National Assessment program (such data will be available in about 3 years, see OME's Social Indicators, 1973, p. 81), it would seem as plausible that quality has gone down as up.

Industries with very rapidly increasing output tend to have better productivity records than do those whose output is increasing less rapidly, and this may account for the relatively better performance of higher education than of elementary and secondary.

5. Perhaps the most careful study of productivity change in education was one undertaken by Woodhall and Blaug[6] for British secondary education in the period from 1950 to 1963. They examined productivity change with respect to three measures of output: (i) numbers of graduates, (ii) numbers of graduates weighted by the number of passing scores they received on standard British examinations, and (iii) numbers of graduates weighted by the relative earnings expected to result from different levels of education the graduates could be expected to attain. Depending on which of these output measures they used, and on exactly how they treated inputs, Woodhall and Blaug obtained a range of estimates for productivity change in British secondary schools. They concluded that, at best, productivity had declined at a rate of 1% per year over the period 1950-1963; it had probably declined more rapidly. In a separate study, Woodhall and Blaug[7] concluded that there had also been a decline in the productivity of British higher education.

The British results suggest that, when quality adjustments are made, the decline in productivity of U.S. elementary and secondary education may be less sharp than the 2.25% per year indicated in (4) above. But even after adjustments, the results will very likely continue to show decline.

6. Why has productivity in education declined? There are two general lines of explanation and each is probably true to some extent. The first explanation emphasizes the lack of incentive for efficiency improvement that managers of public schools may have. Without the carrots and sticks of profits and losses that (supposedly) motivate the private sector manager, it is assumed that a certain flabbiness grows in school system management. The second explanation is that education is intrinsically highly labor intensive and that, therefore, it is unreasonable to expect productivity improvement in

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[6]Woodhall, M., and Blaug, M. "Productivity trends in British secondary education, 1950-1963". Sociology of Education, 1968.

[7]Woodhall, M., and Blaug, M. "Productivity trends in British university education 1938-1962". Minerva, 1965.

education of the sort that more capital intensive industries have experienced. The Office of Finance and Productivity of the National Institute of Education is exploring a range of possibilities for productivity improvement in education; these include efforts both to change the incentive structures for school decision-makers and to develop uses of technology in education that would either improve the quality of output or reduce the rate of increase of costs.

II. NOTES ON EDUCATIONAL PRODUCTION FUNCTIONS

My purpose in these notes is to provide a brief guide to the available literature on educational production functions (epfs). I distinguish among three broad categories of epfs; the distinction depends on the degree of aggregation of the analysis. At the micro level, one is concerned with factors influencing a student's acquisition of particular items of information or concepts. At the meso level one is concerned with factors influencing an individual student's progress through broad curriculum areas. At the macro level one deals with factors influencing the annual numbers of 'graduates' at various levels of a school system. Parallels exist, of course, in other areas of applied production theory where the level of analysis varies from highly aggregate to micro, 'engineering' production functions. Study of micro epfs has principally been the domain of pure psychologists; study of meso epfs the domain of educational psychologists, economists, and sociologists; and study of macro epfs the domain of educational planners. My view is that many of the most important research questions now confronting us lie at the hitherto relatively unstudied micro-meso and meso-macro boundaries. I will thus in these notes attempt to provide references that overview all three levels of epf in order to give background for my conclusions concerning appropriate directions for policy relevant research.

Corresponding to an epf at any given level there is a problem of optimization given input prices and desired level of output (or output price). Educational psychologists view solution of such optimization problems as creating 'theories of instruction', which they contrast with theories of learning; in economics this optimization involves obtaining factor demand functions. I will attempt to provide reference to the problems of optimization associated with each of the three levels of epf. Such optimization is, in a sense, the formal aspect of policy analysis.

Micro-Level Educational Production Functions

Production functions at this level model learning of relatively simple items. A typical paradigm is that of 'paired-associate learning'

in which a single stimulus is to be linked to a single response, for example, in initial reading, the linking of phonemes to graphemes. The micro-epfs that have been constructed to model this type of behavior tend both to be the most sophisticated mathematically of epfs at any level and to provide the best fits of models to data. A standard reference is:

R. Atkinson, G. Bower, and E. Crothers, An Introduction to Mathematical Learning Theory, circa 1967.

A broader range of models is presented in a somewhat more mathematical format in:

D. Jamison, D. Lhamon, and P. Suppes, 'Learning and the structure of information', in J. Hintikka and P. Suppes (Eds.), Information and Inference. Dordrecht: D. Reidel and Co., 1970.

The optimization theories corresponding to various models of paired-associate learning have received increasing attention; a clear and informative overview may be found in:

R. C. Atkinson and J. A. Paulson, 'An approach to the psychology of instruction', Psychological Bulletin, 1972, 78, pp. 49-61.

A more mathematical paper dealing with a specific optimization problem arising in teaching initial reading by computer-assisted instruction is:

V. G. Chant and R. C. Atkinson, 'Optimal allocation of instructional efforts to interrelated learning strands', Journal of Mathematical Psychology, 1973, 10, pp. 1-25.

Meso-Level Educational Production Functions

Production functions at this level are usually simple regressions that have a student's achievement test score on the left hand side and a variety of school, peer, and student background variables on the right hand side. Some of the studies, particularly those using an experimental

methodology, provide only two or three levels for variables on the right hand side, and thus provide only limited information on the epf as a whole. A number of surveys of this research are now available; perhaps the best known is:

H. Averch, S. J. Carroll, T. S. Donaldson, H. J. Kiesling, and J. Pincus, How effective is schooling? A critical review and synthesis of research findings. Santa Monica, The Rand Corporation, 1972.

Averch, et al, synopsize the various meso-epf studies by author; a paper that organizes them by independent variable and includes studies of various educational technologies is:

D. Jamison, P. Suppes, and S. Wells, 'The effectiveness of alternative instructional media: A survey', Review of Educational Research, 1974, 44, pp. 1-67.

Two surveys that emphasize research undertaken outside the U.S. are:

Postlethwaite, T. N. 'A selection from the overall findings of the I.E.A. study in science, reading comprehension, literature, French as a foreign language, English as a foreign language, and civic education', paper presented at the Conference on Educational Achievement, Harvard University, November 1973.

L. Alexander and J. Simmons, 'The determinants of school achievement in developing countries: The educational production function as an analytical tool'. World Bank Staff Working Paper, Draft, July 1974.

The first of these papers surveys results from the mammoth International Study of Educational Achievement (IEA) (if you wish, an international Coleman Report) that was conducted in 28 countries.

The above, then, are a number of surveys of results of epfs at the meso level. What do the surveys conclude? I quote here from the above mentioned paper by Patrick Suppes, Stuart Wells, and me:

Traditional Instruction (JSW, pp. 26 - 27)

"In concluding this survey on the effectiveness of traditional classroom instruction, it seems reasonable to conclude that few variables consistently make a difference in student performance; the 16 variables examined in Table 1 support this generalization. The student-to-teacher ratio is perhaps the most economically relevant variable under a school system's control, and the 23 recent studies summarized in Table 2 strengthen the conclusions of earlier studies that concluded that the student-to-teacher ratio has little influence on student learning. Exceptions to these general conclusions would be that teacher verbal ability appears important in a high fraction of the instances examined, and that small classes seem to improve the cognitive and affective performance of young children. These conclusions do not, however, imply that schools make no difference in the cognitive development of their students; on the contrary, school attendance is clearly important in promoting academic achievement though few studies seem to have examined this issue. What remains to be demonstrated is that variations in traditional school inputs are consistently related to variations in school outputs."

Instructional Radio (JSW, pp. 33 - 34)

"Radio has been used extensively for formal classroom instruction in the United States (more in the past than at present) and elsewhere. There exist, however, only a limited number of good evaluations of the effectiveness of instructional radio. These evaluations indicate that instructional radio (supplemented with appropriate printed material) can be used to teach most subjects as effectively as a live classroom instructor or ITV. Due to the limited number and scope of good evaluations now available, and to the potential economic significance of instructional radio for developing countries, much more research --both survey and experimental--is highly desirable.

Instructional Television (JSW, p. 38)

> "ITV can teach all grade levels and subject matters about as effectively as traditional instruction, though some evidence indicates that it performs relatively better at lower grade levels. A significant fraction of teachers and students have initially negative attitudes toward ITV; these negative attitudes tend to lessen, but not necessarily disappear with time and appropriate administrative behavior. Evaluations that report no significant difference between ITV and traditional instruction are usually based on experimental designs that hold almost everything but the medium constant. It is plausible--though not, to our knowledge, experimentally verified--that attempts to use the distinctive potential of the television medium would result in more systematic findings of significant differences between ITV and alternative treatment groups."

Computer-Assisted Instruction, CAI (JSW, p. 55)

> "As in other methods of instruction surveyed in this report, no simple uniform conclusions can be drawn about the effectiveness of CAI. At the elementary-school level, CAI is apparently effective as a supplement to regular instruction. There are no examples yet of CAI's being introduced with a concomitant change in student-teacher ratio, which would, for example, cover the costs of CAI. At the present time, we can only conclude that CAI can be used in some situations to improve achievement scores, particularly for disadvantaged students.
>
> At the secondary school and college levels, a conservative conclusion is that CAI is about as effective as TI when it is used as a replacement. It may also result in substantial savings of student time in some cases. Since the equal-effectiveness conclusion seems to be broadly correct for most alternative methods of instruction at the college level, there should be in the future increasing opportunities to experiment with selecting the method of instruction in terms of costs, and real opportunities should exist for substituting capital for labor, especially as the relative costs of technology in comparison to labor decline over the next decade."

The methodology of data analysis is, for most of the studies, as I have indicated, multiple regression analysis. The data are most frequently from a single cross-sectional survey (e.g. the IEA study or the Equality of Educational Opportunity (Coleman) Survey) and often it is impossible to specify exactly the school inputs received by any particular student. One is thus taking a snapshot, with the data collection instrument, of a process that inherently evolves over time. A number of papers have attempted

partially to overcome this problem by employing stimultaneous equation estimation techniques to cross-sectional data. Two (of a number) that analyze the Coleman data in this way are:

H. Levin, 'A new model of school effectiveness', in A. Mood (Ed.), Do Teachers Make a Difference? USGPO, 1970.

A. E. Boardman, O. A. Davis, and A. Lloyd, 'A simultaneous equations model of the educational process restructured: Emphasizing the role of college preparatory programs, self-esteem, and the ability to control one's environment'. Paper presented at the North American Winter Meeting of the Econometric Society, San Francisco, December, 1974.

Other papers have attempted to improve on the cross-sectional studies by obtaining longitudinal data. Each of the following three studies has two point in time longitudinal data on achievement (i.e., before and after) plus information on the nature of the instructional inputs received by the students in the interim. I judge studies of this sort as far more credible than results of even the most sophisticated analyses of cross-sectional data.

E. Hanushek, Education and Race. D. C. Heath and Co., 1972. Chapter 3, pp. 35-52.

S. Wells, B. Whelchel, and D. Jamison, The impact of varying levels of computer-assisted instruction on the academic performance of disadvantaged students. Educational Testing Service, Research Bulletin RB-74-20, 1974.

S. Klees, Instructional technology and its relationship to quality and equality in education in a developing nation: A case study of instructional television in Mexico. Ph.D. thesis, Stanford University, 1974; Educational Testing Service, forthcoming. Chapter V, pp. 105-148.

Klees' thesis is the best meso-epf analysis I have seen. His data include before and after achievement measures on each student in several subjects, observation of actual behavior of each student's teacher, and a variable indicating whether the student was exposed to ETV.

One final point that I should mention concerning meso-epfs concerns the role of time in learning. In an intuitively obvious sense, time plays an essential role; however, very few of the available studies include any time variable at all. The following papers do, however, place time variables in a more central role:

Carroll, J.B. 'A model of school learning', Teachers College Record, 64, 1963, pp. 723-733.

Bloom, B. 'Time and learning', American Psychologist, in press.

Wiley. D. E. 'Another hour, another day: Quantity of Schooling, a potent path for policy', School of Education, University of Chicago, 1973.

The optimization aspects of meso-level epfs involve straightforward applications of differential calculus. One highly qualitative and influential discussion of optimization at the meso level is:

J. Bruner, Toward a Theory of Instruction. Harvard University Press, 1966.

Macro-Level Educational Production Functions

The purpose behind construction of macro-level epfs, more frequently called simulation models, is to project the number of enrollees, graduates, and dropouts of an education system as a function of budget levels, dropout rates, and repetition rates. The output of the system in a given year can be considered to be the function that gives the number of school leavers at any given level. The term 'level' here could be understood in terms of cognitive or affective states, but for most purposes it is used to mean grade level. The output function for year t, f_t (g), will, then, give the number of students who leave school in year t after having completed the gth grade. The purpose of a macro epf is, then, to project f_t (g) as a function of budgets and other parameters. The diagram on the following page illustrates the components that go into such models; the epf illustrated there is one that I am preparing for elementary education in Nicaragua.

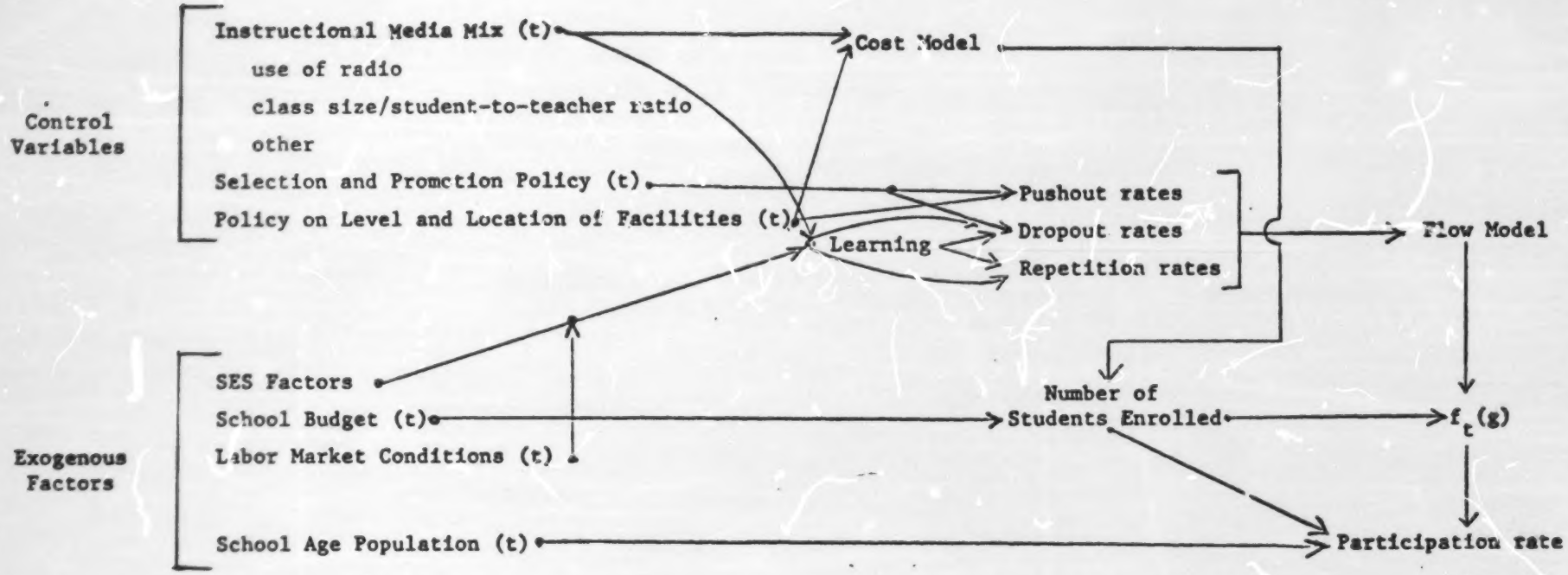

Figure 2: Schematic of Simulation Model for Elementary Education in Nicaragua.

Notes:
1. 't' is appended to a number of variables to indicate time dependence.
2. The term 'pushout' is used to denote individuals forced by the system (purposely or not) to discontinue their schooling.
3. A more refined version of the model will include analysis of teacher training institutions.

The following three documents describe various macro-epf models:

D. Jamison, 'Alternative strategies for primary education in Indonesia: A cost-effectiveness analysis', Research Paper 46, Stanford University Graduate School of Business, 1971.

'The UNESCO Educational Simulation Model', Reports and Papers in the Social Sciences, No. 29, UNESCO, 1974.

Ta Ngoc Chau, Population Growth and Costs of Education in Developing Countries, International Institute for Educational Planning, Paris, 1972.

The optimization techniques used to complement the macro-epf models generally use linear programming. Perhaps the best known of such optimization analyses is:

S. Bowles, Planning Educational Systems for Economic Growth. Cambridge: Harvard University Press, 1969.

Bowles' model maximizes the present value of the difference between benefits in dollar terms (computed as a linear function of the number of graduates at each level) and dollar costs. Another approach is to fix output level and minimize the cost of attaining that output; this is done in the following study:

E. Schiefelbein and R. G. Davis, Development of Educational Planning Models and Application in the Chilean School Reform. D. C. Heath and Co., 1974.

Policy Relevant Research

As I indicated at the outset, I believe important research directions to lie at the boundaries separating the micro and meso levels and the meso and macro levels of epf.

At the micro-meso boundary and in the meso area I expect important research results will emerge from the following:

(i) increased use of the experimental techniques of the micro-level to examine meso-level issues,

(ii) increased use of detailed observation of classroom processes in constructing meso-level epfs,

(iii) increased use of time series information, as is standard at the micro-level, in constructing meso-level epfs,

(iv) increased examination of the role of technology in educational production, and

(v) increased attention to the role of time in learning.

At the meso-macro boundary, I would predict increased emphasis to be placed on examining the effect of school policy variables on other things than learning. These other things might well include dropout propensities, attendance, attitudes, repetition propensity, and the interrelations among learning and these variables. Studies of factors influencing these will, increasingly, be considered within the domain of meso-epf analysis; results of these analyses will be essential in constructing valid macro-planning models.

III. EVALUATION OF THE EFFICIENCY AND PRODUCTIVITY OF EDUCATIONAL SYSTEMS

Much of the recent discussion of accountability in education has as an underlying premise the assumption that improved information and reward structures can contribute to the productivity (or efficiency) of resource use in education. Our purpose in this chapter is to state definitions of these concepts in a way that will provide a coherent terminology for educators and that is reasonably consistent with common usage of these terms in the economics literature.

The discussion begins by defining concepts relating to production and then, using approximately the same categories as Lau and Yotopoulos [1971], defines several separate concepts of efficiency. Productivity is then defined. These definitions assume the existence of market prices for all inputs and outputs. While for the education sector it is reasonable to assume that market prices are available for most inputs, there are no market prices for education's outputs; it is thus necessary to discuss how to modify the concepts of productivity and efficiency when value measures for the outputs fail to exist. These problems relate to the nature of cost-benefit and cost-effectiveness analysis as mechanisms for improving efficiency, and these techniques are briefly discussed.

PRODUCTION POSSIBILITIES

The discussion of production possibilities follows the conventions in regular use in economic analysis and draws in particular on the exposition of Arrow and Hahn ([1971], Ch. 3). Production is carried out by firms,[1] each of

[1] The term "firm" is used here to indicate a unit that makes production decisions; it need not be privately owned. For purposes of this discussion, schools or school districts should be considered firms.

which chooses an activity vector for each time period from within its production possibility set. If there are n commodities in the economy, an activity vector consists of a point in the n-dimensional commodity space; negative components of the activity vector signify commodities used as inputs to the firm and positive components signify outputs of the firm. If the firm is a school district its inputs include use of teacher time and school facilities; its outputs include changes in the distributions of specified student competencies.

It would be ideal, of course, if a firm could choose an activity vector that had all--or almost all--positive components. But the impossibility of creating a product from nothing or even producing a great quantity from very little implies that each firm has constraints on the activity vector it can choose; these constraints are delimited by the firm's production possibility set. The production possibility set for firm f, denoted Y_f, is the subset of the commodity space from which the firm may choose. Different firms may have different production possibility sets because of their ownership of commodities private to the firm (e.g., managerial skill or patents). In what follows, we state certain standard assumptions about the production possibility sets for firms, i.e., for all f: (i) Y_f includes the zero vector[2] and is closed, (ii) Y_f contains no vector except the zero vector without at least one strictly negative component, and (iii) Y_f is convex. Of these assumptions, only convexity might be considered an inappropriate abstraction of the real world.

The relation between the production possibility sets discussed here and the production function concepts frequently used in the economics of education[3] is straightforward. Suppose, for example, that firm f can produce commodity

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[2]The zero activity vector corresponds to the firm's doing nothing; it uses zero inputs to produce zero outputs.

[3]For a discussion of education production functions see Bowles [1970] or Levin [1971]; in practice, educational production functions are the results of regression analyses that have some school output as the dependent variable and various school inputs as the independent variables.

3 from commodities 1 and 2 by the production function $y_3 = g_f(y_1,y_2)$. Then $Y_f = \{(-y_1,-y_2,y_3) \mid y_1 \geq 0,\ y_2 \geq 0, \text{ and } y_3 \leq g_f(y_1,y_2)\}$. Naturally, the restrictions on Y_f that were stated in the preceding paragraph imply restrictions on the possible functions g_f.

If there exist prices for the commodities, these prices can be represented by a vector $p = (p_1,\ldots,p_n)$ in the unit simplex P_n in n-space ($P_n = \{p \mid \sum_{i=1}^{n} p_i = 1$ and $p_i \geq 0\}$).[4] For any given set of prices, then, if firm f chooses an activity vector y_f ($y_f \in Y_f$), the value of y_f will equal $p \cdot y_f = \sum_{i=1}^{n} p_i y_{fi}$, where y_{fi} is the i^{th} component of y_f. Since inputs are negative components of the activity vector, and outputs positive, the value of y_f is the difference between the value of the outputs and the value (cost) of the inputs; it represents the gain or profit -- we will use the term 'gain' because of the public sector nature of schooling -- to the firm of choosing y_f if the prices are p. A central behavioral assumption of economic theory is that firms will choose y_f so as to maximize gain at the existing prices.

ECONOMIC EFFICIENCY

The first concept of efficiency to be defined, that of economic efficiency (or simply efficiency), deals with the extent to which a firm does, in fact, maximize gain. Let $\pi_p(Y_f) = \max_{y \in Y_f} \{p \cdot y\}$; $\pi_p(Y_f)$ is, then, the maximum gain possible for firm f when the prices are p. Convexity and closure of Y_f help assure that $\pi_p(Y_f)$ will have a well-defined maximum, and the inclusion of zero in Y_f assures that $\pi_p(Y_f) \geq 0$. A production decision (activity vector) for firm f, y_f, is economically efficient at prices p if and only if $p \cdot y_f = \pi_p(Y_f)$. If it is assumed that no firm will choose to operate at negative profits, an economic efficiency index can be assigned to choice y_f by taking the ratio of

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[4] Assuming that prices can be represented in P_n is equivalent to assuming that only relative prices matter; it does not matter whether these relative prices are expressed, for example, in dollars or lira.

$p \cdot y_f$ to $\pi_p(Y_f)$. If this index is one or if $\pi_p(Y_f) = 0$, the firm is economically efficient; if it is less than one, it gives an indication of the extent to which the firm is economically inefficient.

Two conceptually distinct sources of economic inefficiency exist. The first of these is sometimes labeled technical inefficiency, for example, by Timmer [1970]. To avoid confusion with a separate concept of technical efficiency, and to accord with other economic usage, the term Pareto-inefficiency is used here. A production decision $y \in Y_f$ is Pareto-inefficient at prices p if and only if there exists a $y' \in Y_f$ such that $y'_i > y_i$[5] for all i and, for some j, $y'_j > y_j$ and $p_j > 0$. What this says is that the firm is Pareto-inefficient if it could have but failed to choose an activity vector with more of a valuable output or less of a costly input, everything else being the same. Timmer [1970] and others have devised ways of empirically measuring Pareto-inefficiency, though this is difficult to do unless there exists a method of ascertaining Y_f without using observable choice data.

The second source of economic inefficiency is frequently labeled allocative inefficiency. Let Y'_f be the set of all Pareto-efficient points in y_f; under reasonably general assumptions there will exist a production function, i.e., a function g_f such that $g_f(y) = 0$ if and only if $y \in Y'_f$. Assuming g_f to be differentiable, the marginal rate of substitution of any two commodities for one another can be computed by implicit differentiation, i.e., one can obtain values for $\partial y_i / \partial y_j$. A Pareto-efficient activity vector is said to be allocatively efficient at prices p if and only if for all i, j ($i, j = 1,\ldots,n$), $\partial y_i / \partial y_j = -p_j / p_i$. If, for example, commodities 1 and 2 were outputs and $\partial y_1 / \partial y_2 > -p_2 / p_1$, the total value of output could be increased by increasing

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[5]The symbols y'_i and y_i represent the i^{th} components of y' and y.

the output of commodity 1 at the expense of having less of commodity 2. Thus a reallocation of output would increase the total value of output and the original situation would have been allocatively inefficient.

At this point three concepts of efficiency have been introduced. A firm is economically efficient at the prevailing prices if it maximizes the gain resulting from the activity vector it chooses from its production possibility set. If it is not economically efficient this may be due either to Pareto or allocative inefficiency. The economic efficiency of a firm, it should be stressed, is defined in terms of the range of options open to it; if that range is narrow for firm f, then f can be economically efficient even though it might use up substantially more resources than firm g in order to produce the same level of output. If so, then in some sense g has access to a better technology than does f. This "access to a better technology" will be termed here "more technically efficient"; this usage corresponds reasonably closely to that of Lau and Yotopoulos [1971].

TECHNICAL EFFICIENCY

Whereas economic efficiency is defined for a single firm in terms of its actual choice in relation to its range of possible choices, technical efficiency is a relation between different firms (or the same firm at different points in time). Firm f is more technically efficient than firm g at prices p if and only if $\pi_p(Y_f) > \pi_p(Y_g)$. That is, firm f is more technically efficient if its potential gain (assuming economic efficiency) exceeds those of g. Firm f is absolutely more technically efficient than firm g if and only if for all $p \in P_n$, $\pi_p(Y_f) > \pi_p(Y_g)$.[6]

There exists an extensive empirical literature concerning measurement of technical efficiency; most of the literature is at a highly aggregated level treating an entire economy as a firm. The purpose of the analysis is generally to examine the extent to which technical efficiency has increased over time; most authors conclude that a substantial fraction of the growth in output per

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[6] If one adds a few technical restrictions, the assertion that f is absolutely more technically efficient than g is equivalent to asserting that Y_g is a proper subset of Y_f, except that both contain 0.

capita over time, in the advanced economies, can be attributed to increasing technical efficiency. Nadiri [1970] and Kennedy and Thirlwall [1972] provide surveys of this literature; the paper by Kennedy and Thirlwall reviews, in addition, studies concerning the sources of increased technical efficiency. Levin [1971] discussed applications of the technical efficiency concept to educational production.

PRODUCTIVITY

Productivity, loosely speaking, is the ratio of output to input, or the amount of output per unit input. It is a frequently used index of how well an economy, industry,[7] or firm is doing. Robinson Crusoe, for example, might well measure his productivity by the number of coconuts he picks per hour. Generally, however, it will be impossible to have the units of productivity measurement be in units of actual output per unit of actual input (e.g., coconuts per hour); multiple outputs from and inputs to the production process preclude this. If a set of weights exists to allow aggregation of outputs into a single aggregate output and input into a single aggregate input, one can again construct a productivity ratio; the weights most typically used are prices and the resulting ratio is the dimensionless number giving value of output divided by value of input. Economists term this number "total factor productivity."

This can be phrased in terms of the notation already developed for a firm's activity vector y_f. We define two new n-dimensional vectors y_f^I and y_f^0 in terms of y_f. The symbol y_f^I is equal to y_f in all negative components and zero elsewhere; y_f^0 is equal to y_f in all positive components and zero elsewhere. Thus $y_f = y_f^I + y_f^0$. Obviously, y_f^I is the vector of inputs to firm f and y_f^0 if the vector of outputs from f. The productivity of an activity vector y_f at prices p is $p \cdot y_f^0 / (-p \cdot y_f^I)$.

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[7] An industry is a group of firms producing approximately the same set of outputs.

The problem with using prices as weights is that if one wishes to compare the productivity of the same firm at different points in time (or different firms at the same time in different countries) there may be different price vectors to choose from.

There are a number of points to be made concerning the relation between productivity and the various notions of efficiency. First, increases in technical efficiency lead to increases in potential productivity. Second, moving from a Pareto-inefficient activity vector to one that is Pareto-superior to it will increase productivity. Third, moving from an allocatively inefficient point to an allocatively efficient one may decrease productivity. This can occur whenever there are decreasing returns to scale.[8] For this reason maximization of productivity (or benefit-cost ratios) is not necessarily desirable. One should be concerned with the difference between benefit and cost, i.e., gain, and and this is reflected in the concept of economic efficiency.

EFFICIENCY IN THE ABSENCE OF OUTPUT PRICES

In capitalist economies, governments usually are involved in productive activities for which no private markets exist; that is, there are no prices for the outputs. Education is a clear example of such an activity. The concept of efficiency carries over to the case when there are no prices for outputs, but it yields fewer results.

When there exist prices for all commodities, a firm's choice of an economically efficient activity vector provides answers to two sorts of questions. First, which commodities should the firm produce and how much of each? Second, which inputs should be used in the production process, and how much of each should be used?

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[8] To continue with the Robinson Crusoe example, if Robinson experiences decreasing returns in coconut picking (i.e., he picks fewer the second hour than the first because he tires), then output per unit of input is highest when he picks few coconuts. If Robinson is hungry, however, the "price" of a coconut relative to the "price" of an hour of his time may be very high; it would be allocatively inefficient for him to pick only a few coconuts.

Without prices for the commodities that are potentially outputs, it becomes impossible to provide an answer to the first question in terms of economic efficiency. It does remain possible, however, to provide an answer to the second question.

Consider a strictly positive vector y' in the n-dimensional commodity space: y' may be viewed as the target output vector, i.e., the vector that the firm--for whatever reason--has decided to produce.[9] The economically efficient way for firm f to meet this demand at prices p is to choose a vector $y_f \in Y_f$ (if one exists) such that $y_f^0 \geq y'$ and $-p \cdot y_f^I$ is minimized. (Here, the sign $\geq$ between vectors means greater than or equal to in every component and p is the n-dimensional vector whose non-zero components are the prices of the commodities for which prices exist. The symbols y_f^0 and y_f^I represent, as before, the vectors yielding the outputs and inputs corresponding to activity vector y_f.) This definition of economic efficiency simply asserts that the efficient choice to make is the one that minimizes cost subject to the constraint that output reach at least the specified level y'. The definitions of Pareto-efficiency, allocative efficiency, and technical efficiency also extend in the obvious way to the case where prices for the outputs fail to exist.

The definition of productivity fails, however, unless one has some arbitrary weighting scheme to aggregate the outputs into a single number or unless there is only one component of output. The previous definition of productivity can easily be extended to cover either of these cases.

COST-BENEFIT ANALYSIS AND COST-EFFECTIVENESS ANALYSIS

In the preceding discussion there has been no mention of how the firm knows what its production possibility set (production function) is or how, if it did know it, it would be able to compute its optimal choice. These are the tasks of cost-benefit analysis and cost-effectiveness analysis. In the case where

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[9] y' need not be strictly positive, though that assumption simplifies the exposition here. In education, however, the authorities may feel constrained to use at least a minimum amount of a particular input, for example teachers, for political reasons.

no prices exist for outputs, cost-effectiveness analysis attempts to provide an answer to the question of how total cost and optimal choice of inputs vary as a function of prices and the target output y'. Cost-benefit analysis includes cost-effectiveness analysis but goes beyond it in attempting to assign some measure of value (benefit) to the various possible y's. These benefits can then be compared to costs in coming to a decision concerning the optimal activity vector. In the case where prices do exist for outputs, cost-benefit analysis and cost-effectiveness analysis are similar; both are concerned with ascertaining how total gain and the optimal levels of the inputs and outputs vary with price.

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